IMPACT OF THE BURIED-VALLEY GEOMETRY ON THE GROUNDWATER FLOW: A FINITE-ELEMENT NUMERICAL MODEL

Kazimierz Burzyński & Andrzej Sadurski

Wydział Hidrotechniki Politechniki Gdańskiej, ul. Majakowskiego 11, 80-952 Gdańsk-Wrzeszcz


Abstract: A theoretical analysis of the groundwater flow, undertaken on the basis of a numerical model using the finite-element method, reveals a strong variability of the velocity and residence time within the aquifers of the buried Pleistocene valleys in the Gdańsk region. The estimation of a unit water-portion trajectory allows one to obtain local residence times of groundwater, to spot the places where the oldest water can be found, as well as to describe a vertical change of the groundwater flow in the cross-section of an aquifer.

Key words: groundwater flow, hydrogeochronology, coastal aquifer, finite-element method.

Manuscript received January 1986, accepted March 1987

INTRODUCTION

Occurrence of salty groundwater in the buried Pleistocene valleys along the coastal area of the Baltic Sea in Poland has been proved by the hydrogeological survey made during the last decade. The problem of the origin of this salty groundwater has been analysed more extensively by Kozerski (1983), Sadurski (1984), Kozerski & Kwaterkiewicz (1984) and Kwaterkiewicz & Sadurski (1986).

Apart from a possibility of present-day seawater encroachment into the aquifers, the relic sea-water of the Holocene or even Eemian transgressions might have persisted in the bottom parts of the Pleistocene valleys. Such possibility is indicated by the results of several studies of the Quaternary deposits along the coastal region (Mojski, 1979; Makowska, 1982; Kondratiene & Gudelis, 1983).

Kleczkowski (1963) proved the important role of wash-out processes in the young sediments of the coastal aquifers on the example of Hopei Plain (North China). According to him, the relic sea-water could persist in the Hopei aquifers in some low-permeability strata of low hydraulic gradient.

Natural isotopic composition is now commonly used in investigations of
the groundwater origin and age (Hanshaw & Back, 1974; Dowgiałło, 1976; Herraez & Llamas, 1983). It allows one also to infer on groundwater circulation in the aquifers of regional extent, and on the rate of the groundwater flow.

The present authors suggest another approach, i.e. the application of the finite-element method to the above-mentioned problems. Analysis of the groundwater unit volume flow made it possible to calculate the flow time in the modelled part of the aquifer over the distance from the entrance of the water to its exit. If the assumed entrance of the unit volume of groundwater is approximately in the recharge zone of the aquifer one could calculate the age of groundwater. The term “residence time” has been used in the paper to denote period when groundwater remains within the modelled part of an aquifer (Silar, 1982).

The analysis of the groundwater flow carried out in this paper can be of use in designing and interpreting the hydrogeochronological investigations using natural isotopes in water.

HYDROGEOLOGICAL SETTING OF THE ANALYSED AQUIFERS

The analysed region extends over 100 km from the Vistula River Delta in the south, through Gdańsk and Gdynia towns to the Żarnowieckie Lake trough in the north. It is a belt situated on the border of the moraine hills of the Kashubian Lake District and the coastal lowlands (Fig. 1).

The Delta surface deposits are composed of fluvial sediments whose thickness amounts to 30 metres. The roof of this sequence consists of peats and muds and fine sands (Figs. 2, 3). Similar sediments occur in the Żarnowieckie Lake trough. The peats and muds represent in part a Recent lacustrine accumulation (Fig. 4).

Below the Holocene sediments there appear fluvioglacial sands and gravels interbedded with glacial tills and silty sands. These sediments belong generally to the North Polish Glaciation (Vistulian). One can distinguish the deposits of Mid- and South Polish Glaciations (Saalian and Elsterian) in the deeper part of the Quaternary profile of the Baltic Sea coast. The biggest thickness of the Quaternary sediments occurs in the buried Pleistocene valleys. These valleys are typical of the described area. They were re-exarated during succeeding glaciations and were filled with loamy, silty and sandy deposits of the fluvioglacial origin and with interglacial sediments.

The Pleistocene erosion reached the Mesozoic strata in the northern part of the Vistula River Delta only. In the other parts of the Gdańsk Region there is a continuous cover of Tertiary sediments developed as a typical brown-coal facies.

The Upper Cretaceous deposits occur generally at a depth of 100 m below sea level. They consist of geizes, marls, glauconitic fine sands and sandy
limestones of Campanian age. The Cretaceous aquifer are glauconitic fine sands, lying 50 metres beneath the roof of the Mesozoic strata and locally it is connected with fissures in the geizes and marls. The water of the Tertiary aquifer is of minor importance because of the low-permeability deposits and low, indirect recharge. The chemical composition of this water is similar to the Quaternary water of $\text{HCO}_3^- - \text{Ca}$ type.
Regardless of their origin, the conditions of the groundwater occurrence in the water-bearing Pleistocene series are similar. They are common for sea bars, the Vistula River Delta, Kashubian moraine plateau, Żarnowieckie Lake trough, the Reda ice-marginal valley and the so-called "marine terraces". The water-bearing fluvioglacial sands of transmissivity up to 1000 m²/day, can be

---

Fig. 2. Geological cross-section of the buried valley near Pruszcz Gdański. 1 — marl and limestone; 2 — loam and mud; 3 — silt; 4 — till; 5 — sand; 6 — groundwater table and flow directions; 7 — drill holes; $k_i$ — hydraulic conductivities

Fig. 3. Geological cross-section of the buried valley near Tczew. Explanations as to Fig. 2
found at depth down to 35—40 metres. There exist low-permeability sandy series beneath the mentioned aquifers, especially in the buried Pleistocene valleys.

Salty groundwater has been found in some of these valley in the deeper parts of the aquifers. The salty water is of Cl—Na type and has the same chemical composition as relic sea water (Kozerski, 1983; Kozerski & Kwaterkiewicz, 1984; Kwaterkiewicz & Sadurski, 1986).
The regional groundwater flow is from the moraine hills of the Kashubian Lake District to the discharge area situated on the Vistula River delta plain, and in the coastal lowlands around the Gulf of Gdańsk.

The presence of deep Pleistocene valleys situated along the border of moraine hills in the analysed region should influence the regional flow system. The more detailed analysis of the groundwater flow perpendicular to the Pleistocene valleys was carried out on four examples shown in Figs. 2—5. The presented geological cross-sections indicate big variability of thickness and hydraulic conductivity within the considered aquifers. Transmissivity and hydraulic conductivity of the studied aquifers were taken from pumping tests carried out by the Geological Company of Gdańsk. Effective porosity of the water-bearing strata was estimated from particle-size distributions (Kozerski, 1971). The hydraulic conductivity of semi-permeable deposits, e.g. silty sands and loams was obtained from the literature (Kováčs, 1981; Dąbrowski, 1982; Kerkis, 1975).

Simplification of the hydrogeological conditions of the buried valley aquifers under consideration has been based on the geological maps and cross-sections. Boundary conditions of the first or second order: \( H = \text{const.}, \quad \frac{\partial H}{\partial n} = 0 \), were taken for all the analysed examples. The hydraulic gradient of groundwater is very low in the studied aquifers and does not exceed 0.001.

**MATHEMATICAL MODEL OF GROUNDWATER FLOW**

Because of the limited possibilities of obtaining analytical solutions due to complicated geometry of the area and complex geological structure, the groundwater flow in the described aquifers has been analysed by means of an approximate mathematical model. Assuming steady-state water flow in a water-bearing system saturated with a non-compressible fluid, it is possible to use Boussinesq’s equation (Polubarinova-Kočina, 1962; Bear, 1979).

If the flow is assumed to occur in gravitational field only, under isothermal conditions, and if the forces of chemical nature are neglected, the fluid potential can be defined as:

\[
\varphi = x_i + \frac{p}{\rho q} g
\]

where: \( x_i \) — distinguished vertical direction of axis of the adopted system of coordinates,
\( p \) — pressure of water,
\( \rho \) — fluid density,
\( g \) — acceleration due to gravity.

Under steady-state flow conditions in a two-dimensional model, where \( z \) and \( x \) are vertical and horizontal directions respectively, within area \( \mathcal{D} (\mathcal{D} \in (x, z)) \) with a boundary \( S \), Boussinesq’s equation takes the form:
where: \( K_x, K_z \) — coefficients of filtration tensor,
\( b \) — width of the groundwater flow system, transverse to the \( x \) direction,
\( \bar{R} \) — mean source term.

The shape of the system boundaries is often determined by the shape of impervious layers or by the position of the water table.

Relation (2) is, with regard to \( \phi \), a non-linear partial differential equation of elliptic type. The non-linearity of the equation results from the fact that there occur unconfined aquifers and part of the boundary \( S \) is unknown. However, it is obvious that the surface is a boundary streamline under steady-state flow conditions. Points \( P \) lying upon this surface require simultaneously two boundary conditions to be satisfied, namely:

\[ \phi_p = Z_p \]

and

\[ \frac{\partial \phi_p}{\partial n} = 0. \]

For the purpose of approximation of the unknown flow parameters in Eq. (2), advantage has been taken of the finite-element method with the application of Galerkin's procedure (Zienkiewicz, 1972; Gallagher et al., 1975; Connor & Brebbia, 1976).

The values \( \{\phi\} \) searched here (\( \{\cdot\} \) —denotes the column matrix) are approximated by polynomial \( \{\hat{\phi}\} \) of the form:

\[ \{\hat{\phi}\} = [N_i] \{\phi_i\} \]

where: \([\cdot]\) — row matrix,
\( N_i \) — basis function further referred to as shape functions,
\( \phi_i \) — unknown coefficients.

The shape functions, for the grid of triangular elements adopted in the solution, are linear functions satisfying the following conditions:

\[ N_i(x_m, z_m) = \delta_{lm}, \quad (l, m = 1, 2, 3, \ldots, NE), \]

where: \( (x_m, z_m) \) — coordinates \( m \), of the grid nodes,
\( \delta_{lm} \) — Kronecker's delta.

In the case considered in this paper, for a two-dimensional groundwater flow system \( \mathcal{D} \) covered by \( NE \) elements, orthogonal conditions can be written according to Galerkin's procedure (Zienkiewicz, 1972).

To eliminate derivatives of the second order from the equation used, Green's transformation has been employed. Discretization of the aquifer is carried out in such a way that the hydraulic conductivities are kept constant.
for respective elements (integration fields), although they may vary from element to element.

The algorithm of the solution assumes that there is no flow across the adopted initial water table, what is subsequently iteratively corrected until condition \(|\varphi_m - z_m| < \varepsilon\) is satisfied for all nodes situated upon the surface. \(\varepsilon\) — is the adopted test value of having completed the calculations.

The next step is the determination of hydraulic head vector for all nodes within the solution area. Knowing this vector in zone \(\mathcal{D}\) it is possible to calculate the velocity field \(\vec{v}_e(u^e, v^e)\) inside elements built on three nodes, by means of the following numerical Darcy’s formulae:

(a) horizontal velocity of groundwater flow:

\[
u^e = -\frac{K^e_z}{n^e \cdot 2 \cdot F^e} \sum_{l=1}^{3} b_l \varphi_l,\tag{7}
\]

(b) vertical velocity of groundwater flow:

\[
u^e = -\frac{K^e_x}{n^e \cdot 2 \cdot F^e} \sum_{l=1}^{3} c_l \varphi_l,\tag{8}
\]

where: \(K^e_z, K^e_x\) — hydraulic conductivity in \(z\) and \(x\) directions respectively, \(\varphi_l\) — flow potential in a node \(l\) of an element \(e\), \(F^e\) — surface of triangular element obtained from discretization of the aquifer, \(b_l, c_l\) — coefficients in the polynomial:

\[N_l = a_l + b_l x + c_l y,\]

\(n^e\) — effective porosity.

By introducing streamline \(\Psi\), as a family of holomorphic lines in relation to \(\varphi\), one can repeat the previously applied procedure, by formulating boundary conditions of the 1st or 2nd order. The calculated field of the flowing water enables one to estimate the time during which the unit volume of water remains in the analysed flow system. It is the residence time \(t\), obtained as \(t = t_p - t_0\), where: \(t_0 = 0\) for the starting point \(x_0, z_0\) situated on the edge of the system and \(t = t_p\) at the time when the unit volume of water leaves the system.

The calculation procedure outlined above is utilized by program in Fortran. The calculations were carried out by means of ODRA-1305 computer operating in George-3E system.

**RESULTS AND DISCUSSION**

Using the above procedure, the following values were obtained: groundwater table in the part of the considered aquifers, piezometric head in each net node, components of the velocity vector of groundwater flow in each net element, flow ratio in net nodes and groundwater stream line. In the next step, residence time of water was calculated using stream lines, velocity of groundwater flow and effective porosity of the aquifers.
Results of the calculations are presented for four chosen examples and shown in Figs. 6A—9A. The dashed lines in these figures describe the volume of flow in percents. The dotted lines illustrate equal residence time \( t \), which on the left-hand border of the model was taken as \( t = 0 \). The values of these isochronic lines are given in years. As it is evident from the mentioned figures, the longest residence time (the oldest groundwater) occurs in the bottom part of the aquifers in the several-metre thick zone where the flow represents only an insignificant percentage of the total groundwater stream.

Calculation results referring to the buried valley near Pruszczy Gdański are given in Fig. 6. The aquifer system consists of four strata of various permeability. One can observe the fifty-fold variation of residence time of water in the cross-section of this aquifer. 90\% of the groundwater stream covers the distance of 1300 metres in 20 years, whereas 10\% of its volume within the bottom stratum flows very slowly and there can be found places of a residence time longer than 1000 yrs (Fig. 6). There is a place in the bottom part of the analysed aquifer in the cross-section II with very long residence time and therefrom one can deduce about the oldest groundwater here.

A long residence time of water has been obtained with regard to the buried valley situated near Tczew (Fig. 7). The maximum residence time for the modelled part of the aquifer is over \( 2 \times 10^4 \) years. 80\% of the stream volume in analysed water-bearing system flows across the studied segment in about 470 yrs. The variability of the residence time is forty times in this case (Fig. 7A). It results from the fact that approximately 1\% of the stream volume is filtrated through low-permeable deposits which fill the discussed Pleistocene valley.

The results of solving the groundwater flow equation for aquifer referred to as “marine terrace” in Gdańsk-Oliwa, are given in Fig. 8. Some 90\% of the groundwater stream volume concentrates within the layer just under the surface with the highest transmissivity. The difference between the shortest and the longest residence times amounts 100 yrs. in the cross-section II. The maximum flow duration at a distance of 2000 metres is ca. 140 yrs.

The flow ratio in the buried valley of the Żarnowieckie Lake trough has been obtained for a modelled segment of the aquifer, 1200 metre long. The stream lines and the isochrones of residence time are presented in Fig. 9. The longest residence time in the zone beneath low-permeable deposits of the lake reaches 200 years and the minimum residence time in the coastal zone is approximately 25 years.

The flow ratio in the buried valley of the Żarnowieckie Lake trough has unit width made it possible to trace the vertical constituent of filtration \( v_p \). It has a significant influence upon the flow intensity in the anisotropic system and in the case of an evident change of thickness of the strata. Differentiation of \( v_p \) within respective strata and identification of places of intensive or sluggish exchange of water would not be possible using a flat model. In the considered cases 80 to 90\% of groundwater of the upper parts of the regional streams have the same local flow time. These parts of streams belong to the intensive circulation of groundwater.
Fig. 6. Results of the computation of groundwater flow and the residence time of water in the buried valley aquifer near Pruszczy Gdańsk. A — graph of the residence time change with a depth of the aquifer; B — discretized aquifer with triangle grid of NE number of elements; 1 — flow lines of groundwater, the percentage of groundwater flow volume is indicated in the rectangle; 2 — isochronic line of equal residence times (value is given in years); 3 — sections for graphs of residence time differentiation given in A; $k_i$ — hydraulic conductivities as in Fig. 2.
Fig. 7. Results of the computation of groundwater flow and the residence time of water in the buried valley near Tczew. $k_i$ — hydraulic conductivities as in Fig. 3. Explanations as to Fig. 6
Fig. 8. Results of the computation of groundwater flow and the residence time of water in the coastal-terrace aquifer near Gdańsk-Oliwa. $k_i$ — hydraulic conductivity as in Fig. 5. Explanations as to Fig. 6.
Fig. 9. Results of the computation of groundwater flow and the residence time of water in the buried valley in the Żarnowieckie Lake trough. $k_i$ — hydraulic conductivities as in Fig. 4. Explanations as to Fig. 6.
A large differentiation of vertical groundwater flow results not only from the facies variability of sediments but also from the geometry of stream. The latter is controlled by the deep buried valleys in the studied area. Therefore, one can deduce that old water may occur at the bottom of an aquifer and can be preserved in the Pleistocene erosive depressions, within the so-called hydrogeological traps.

**CONCLUSIONS**

The calculations of groundwater flow according to the proposed numerical technique enable one to estimate the value of the residence time and vertical differentiation of filtration in the modelled aquifers. One can infer on the basis of the calculated residence time that the buried-valley aquifers in the Gdańsk region carry groundwaters of different ages. It agrees with the studies on the chemical composition of groundwater here (Kozerski, 1983; Sadurski, 1984; Kwaterkiewicz & Sadurski, 1986). One can presume that the salty relic water of marine origin may have persisted till present in the deeper parts of the buried valleys situated perpendicularly to the flow direction. The fifty-fold differentiation of the residence time and the “age” of groundwater of over 1000 years were obtained in the modelled segment of the aquifer on the distance of some kilometres.

Such a low groundwater exchange in the buried valleys in the coastal zone might have locally protected the old sea-water here. This may be water from the Littorina (Postglacial) or even from the Eemian transgression. The removal of the relic salty groundwater by infiltrational waters during “wash-out” processes in the analysed aquifers takes a long time of thousands of years.

**REFERENCES**


Streszczenie

WPŁYW GEOMETRII POGRZEBANYCH DOLIN NA PRZEPŁYWY WÓD PODZIEMNYCH:
MODEL NUMERYCZNY METODĄ ELEMENTÓW SKOŃCZONYCH

Kazimierz Burzyński & Andrzej Sadurski

W osadach kenozoiku Polski północnej spotykane są struktury erozyjno-
akumulacyjne zaliczane do tzw. pogrzebanych dolin, które stanowią strefę tranzysty wód podziemnych odpływających z wysoczyzn pojeziernych do pasa nadmorskich nizin (Fig. 1). Metodą elementów skończonych uzyskano szereg rozwiązań równania Boussinesqa dla przypadków dwuwymiarowego przepływu w profilu pionowym wymienionych struktur. Wyznaczone pola prędkości rzeczywistej płynącej wody pozwoliły na oszacowanie czasu przebywania cząstki wody w analizowanym systemie. Jest to czas lokalny \( t \) przyjmujący wartość \( t_0 \) dla punktu początkowego \((x_0,z_0)\) leżącego na wejściu do systemu oraz wartość maksymalną \( t = t_p \) w momencie opuszczenia systemu. Wyznaczone trajektorie cząstek wody odpowiadające ortogonalnym do linii ekwipotencjalnych liniom prądu (Fig. 6—9) są słuszcze tylko w zakresie laminarnego przepływu. Podane w rozwiązaniu prędkości ruchu wody odnoszą się do warunków izotermicznych w polu grawitacyjnym. Pominięte zostały siły
naturny chemicznej i siły powierzchniowe na granicy faz. Stąd w obszarach o współczynniku filtracji \( k < 10^{-2} \text{ m/24 h} \) i niskich gradientach hydraulicznych mogą wystąpić odchylenia od liniowego prawa filtracji i podane wartości czasu lokalnego są prawdopodobnie zaniżone.

W rozpatrywanych przypadkach 80—90% objętości strumienia wód podziemnych wykazuje ten sam czas lokalny przepływu. Do 20% objętości wód w strefie przyspągowej natomiast charakteryzuje się znacznie wydłużonym (do kilkudziesięciu razy) czasem przepływu. W utworach przypowierzchniowych czas lokalny wynosił średnio kilkudziesięć lat, a w warstwach wodonośnych przy ich spągu czas przepływu — „wiek wód” — przekraczał maksymalnie 1000 lat. Na tak duże zróżnicowanie czasu przepływu wpływa nie tylko mała wodoprzepuszczalność warstw, lecz także geometria analizowanych struktur. Obliczenia stratyfikacji przepływu, a tym samym zróżnicowania czasu lokalnego wód w profilu pionowym, mogą być przydatne w lokalizowaniu miejsc poboru prób do badań izotopowych i oznaczaniu wieku bezwzględnego wody.