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Da Vinci's *Paleodictyon*: the fractal beauty of traces

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ABSTRACT :

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The origins of ichnology are located in a land of convergence between Art and Science, in a historical period – the Renaissance – during which the scientific method had its birth. Trace fossils were studied and graphically represented by preeminent naturalists such as Leonardo da Vinci, Konrad Gesner, Johann Bauhin and Ulisse Aldrovandi – who defined ichnofossils as "exceptionally beautiful".

In this study, the representation of trace fossils in the Renaissance is explored by employing a method widely used in studying visual perception – fractal geometry. In particular, this paper focuses on the reasons for the aesthetic appeal of traces and proves that (1) the aesthetic perception of traces is closely tied to their fractal dimension, and (2) many traces are aesthetically appealing because they have fractal behaviour.

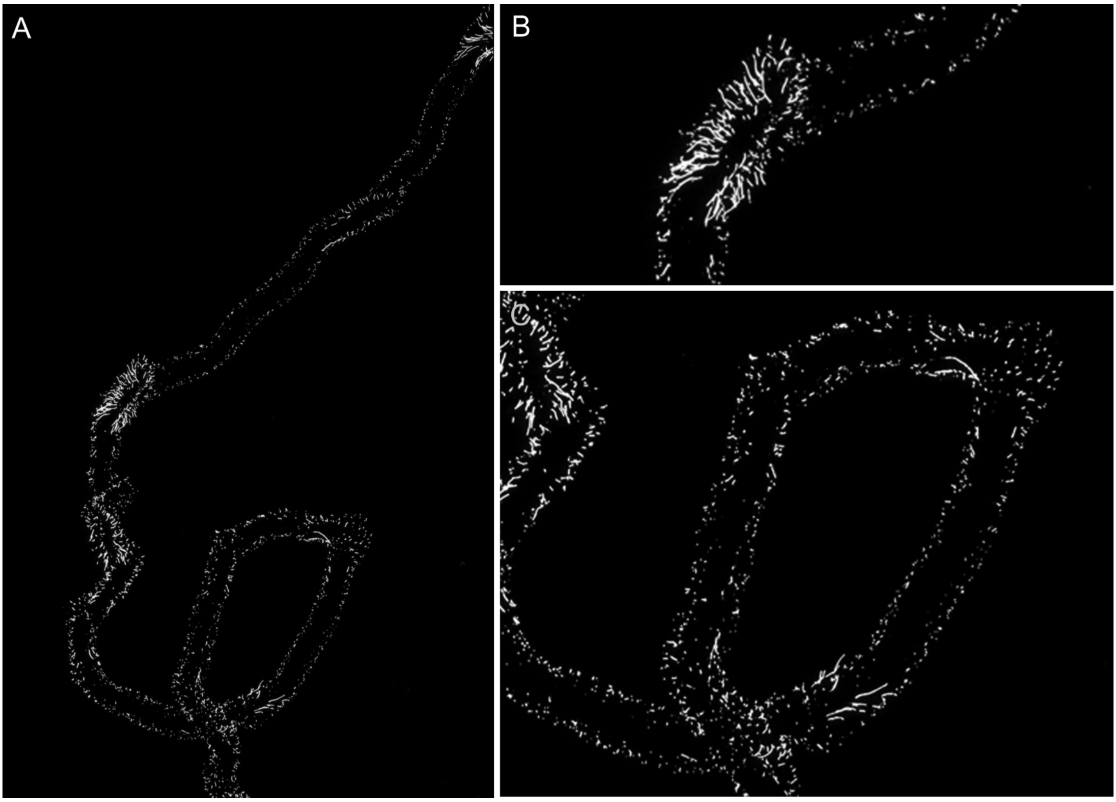
In particular, graphoglyptids and chondritids display significant fractal-like features that are linked with their constructional program and function. Such fractal traces are hierarchically structured and their whole geometric structure can be regarded as an expression of self-organization processes producing correlations between different orders of scale.

Key words: History of science; Ichnology; Fractal, Leonardo da Vinci; Extended organism.

INTRODUCTION: THE AESTHETIC APPEAL OF TRACES

> The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living.
>
> Henri POINCARÉ (1908)

Ichnology and Art are closer than at first appears. The most famous example is, without any doubt, Adolf Seilacher's travelling exhibition 'Fossil Art' (Seilacher 2007), wherein trace fossils are presented for their aesthetic value. Another contemporary example is provided by Martin Prothero, an artist who said, "As an artist, I can think of no better way of representing a wild creature, than letting it represent itself" (Baucon 2009a). Indeed, Prothero realizes trace-centred artworks by recording traces of invertebrates and small vertebrates with a peculiar technique (Text-fig. 1). In addition, Prothero uses and manufactures cameras with biogenic borings as pinholes. Similarly, Pamela Cole uses insects as living paintbrushes by spreading non-toxic paints on their limbs. Traces and footprints are a recurring element of the traditional aboriginal art of Australia and are also featured in the works of several contemporary artists. For instance, Daymirringu Malang has devoted a series of paintings to *luku,* which means foot, footprints, tracks, traces (Baucon 2009a).



Text-fig. 1. Martin Prothero is a contemporary artist who is aesthetically driven by traces. A – Martin Prothero, *Centipede*. B, C – Details of *a*. Note the fine details obtained with the 'carbon coating technique'. The artist coated a glass surface with carbon, and then left the prepared glass outdoors; in this case, a centipede left its trackway on the surface

In addition, the Russian animator Mikhail Aldashin has represented fanciful burrow architectures in his short movie *The Other Side*. The aesthetic charm of traces is not limited to these contemporary examples, for the roots of ichnology lie between Art and Science. The study of traces began in the Renaissance with the ichnological drawings of Leonardo da Vinci, Konrad Gesner, Johann Bauhin and Ulisse Aldrovandi (Baucon 2008a, b, 2009b). Starting from these examples, I will deal with the aesthetics of traces following three points:

1. Ichnological illustration in the Renaissance
2. Aesthetic and cognitive psychology of traces: Why are they visually appealing?
3. Fractal geometry in the aesthetic and scientific study of traces

THE STUDY OF TRACE FOSSILS IN THE RENAISSANCE

Leonardo, the founding father of ichnology

It is not an easy matter to find historical references in ichnology before the 19th century, a period of widespread scientific interest in trace fossils (Osgood 1970, 1975; Lockley 2002; Cadée and Goldring 2007). During the early 1800s, the first pioneering studies on vertebrate footprints were conducted by Buckland, Sickler, Kaup and Hitchcock (Sarjeant 1987; Lockley 2002). In the same period invertebrate trace fossils received significant attention, even if they had a botanical interpretation. Correct interpretations of invertebrate trace fossils were exceptional (e.g. Salter 1856). Accordingly, Osgood (1970) defined this stage of the history of ichnology the 'Age of Fucoids' from the term used to name plantlike traces (e.g. *Chondrites)*. The Age of Fucoids came to an end when Nathorst (1881) highlighted the similarities between 'fucoids' and various kinds of recent traces.

These elements point to the 19th century as the dawn of ichnology, but recent studies have moved the goalposts of ichnology back to the Renaissance ('Age of Naturalists'; Baucon 2008a, b, 2009b). In fact, Leonardo da Vinci left us a wealth of ichnological writings within his notebooks wherein he dealt with bioerosional and bioturbation traces, both modern and recent. Leonardo's ichnological notes were written to support his theory on marine body fossils and sedimentary geology. In the Codex Leicester, Leonardo addressed a problem that tormented his contemporaries: Why seashells are found on mountaintops (*li nichi, che* [...] *lontano da li mari, in tanta altezza si vegghino alli nostri tempi* – “the shells [...] that can be seen far from the sea and at such heights”; Codex Leicester, folio 10v). In da Vinci’s time, many intellectuals proposed an inorganic origin for fossils and believed they were natural curiosities that grew in the ground spontaneously. Other intellectuals described the Deluge as a geological agent and believed as a matter of faith that marine shells were transported by the biblical Flood (Vai and Cavazza 2003). Leonardo refuted both Inorganic and Deluge theories, and among various arguments he cited ichnofossils. Leonardo even took borings into account to demonstrate the organic origin of bioeroded fossils:

“[the Inorganic Theory is not true] because the trace of the animal’s movements remains there on the shell which is consumed by the animal as a woodworm on the wood […]”

Ancora resta il vestigio del suo andamento sopra la scorza che lui già, a uso di tarlo sopra il legname, andò consumando [...].

(Leicester Codex, folio 9v)

Leonardo reported the provenance of these bioerosional trace fossils:

“The hills around Parma and Piacenza show abundant molluscs and bored corals still attached to the rocks. When I was working on the great horse in Milan, certain peasants brought me a huge bag full of them.”

Vedesi in nelle montagnie di Parma e Piacentia le moltitudini di nichi e coralli intarlati, ancora appiccicati alli sassi, de’ quali quand’io facevo il gran cavallo di Milano, me ne fu portato un gran sacco nella mia fabbrica da certi villani.

(Leicester Codex, folio 9r)

Nowadays “the hills around Parma and Piacenza” are still known for their geological and palaeontological heritage. In particular, the ichnological record of this area is consistent with Leonardo’s words: mollusks with borings are very common, and *Oichnus, Maeandropolydora* and *Entobia* are among the commonest bioerosional ichnogenera (Savazzi 1981; Dominici 2001).

In his systematic dispute against the Inorganic and Deluge theories, Leonardo also faced bioturbation structures. The Leicester Code reports some neoichnological observations on a certain species of mollusk that “does not swim, but makes a furrow in the sand and crawls by means of the sides of the aforementioned furrow” ([…] *perchè no nota, anzi si fa un solco per l’arena mediante i lati di tal solco ove s’appoggia, caminerà* […]).

Nevertheless, what is more surprising is his discussion on bioturbation trace fossils:

“Among one and another rock layers, there are the traces of the worms that crawled in them when they were not yet dry.”

Come nelle falde, infra l’una e l’altra si trovano ancora gli andamenti delli lombrici, che caminavano infra esse quando non erano ancora asciutte.

(Leicester Codex, folio 10v)

This excerpt from the Leicester Codex is an exceptional act of synthesis that utilises several ichnological principles that are still valid today:

Tracemakers: Leonardo understood that certain trace fossils were produced by wormlike organisms (*lombrici,* ‘worms’). It is worthy of note that the interpretation of invertebrate trace fossils has been problematic even in later times. Leonardo surpassed, in just one step, the ‘Age of Fucoids’ of the 19th century.

Toponomy: Leonardo described a common preservational style of traces: “among one and another rock layers”, that is, semirelief preservation.

Taphonomy: Da Vinci understood diagenesis of sedimentary layers and taphonomy of trace fossils (“worms that crawled in them when they were not yet dry”). Leonardo’s knowledge of diagenesis and taphonomy is confirmed by Ligabue (1977), Vai (1993) and Vai and Cavazza (2003).

Traces as palaeoenvironmental tools: For Leonardo, certain trace fossils prove the marine origin of rock strata.

Unfortunately, these observations did not influence the academics of the time, because Latin was the language of science and Leonardo wrote in Italian using mirror-writing.

The Mona Lisa of ichnology: Leonardo’s *Paleodictyon*

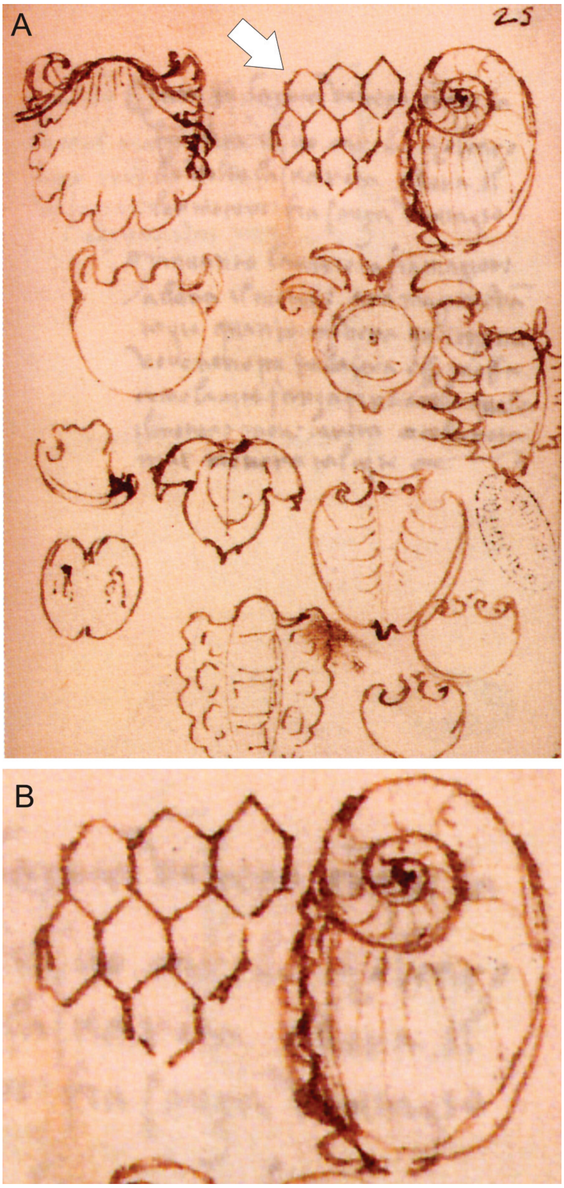
Leicester Codex is not the only evidence of Leonardo’s ichnological interests: The folio 25r of Codex I retains a *Paleodictyon* hand-drawn by da Vinci itself (Text-fig. 2). Unfortunately the picture was not accompanied by any commentary.

The drawing constitutes a hexagonal pattern that fully corresponds to the diagnosis of *Paleodictyon*: a burrow system composed of a hexagonal mesh (Uchman 1995). Despite the clear similarities with *Paleodictyon,* we must first exclude the possibility that it is a hexagonal pattern related to a geometrical or technical study.

The structure is illustrated together with various mollusc shells, most probably body fossils (Ligabue 1977). This suggests that the hexagonal mesh is a geological object.

The hexagonal mesh could represent a honeycomb, but in this case its co-occurrence with mollusc shells would be unusual. Moreover, Leonardo never associated shells and honeycombs in his manuscripts, while in the Leicester Codex (postdating Codex I) he established a conceptual link between fossil shells and trace fossils.

If the structure had been a geometric or technical study, it would have been accompanied by notes in the margins (as in the most other manuscripts of Leonardo; see Codex Forster I, folio 13). For this reason it is highly likely that folio 25 was devoted to 'interesting geological objects', perhaps for use in heraldry.



Text-fig. 2. Leonardo's *Paleodictyon*. A – General view of the folio 25v, where Leonardo collected several 'geological objects'. B – Detail of Leonardo's *Paleodictyon*

Da Vinci was looking in the right place: In the Leicester Codex he admitted to having given special attention to semireliefs (hyporelief is the typical preservation of *Paleodictyon*; Uchman 1995).

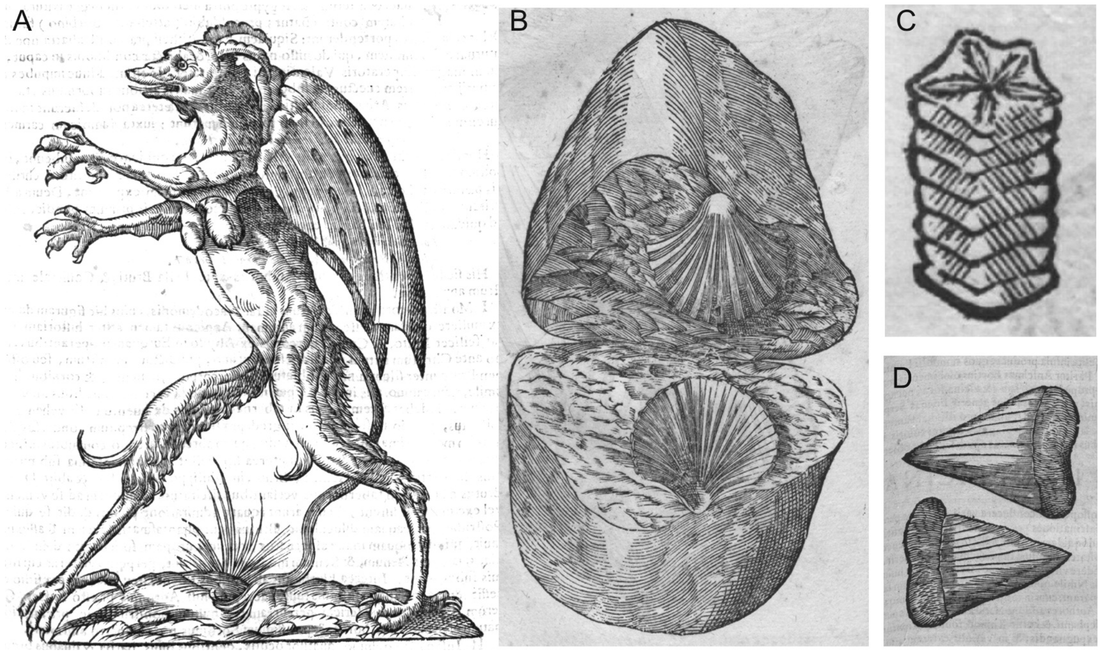
Leonardo travelled and studied in detail the most representative ichnosites of the Italian Apennines, many of which are known for their graphoglyptids (Uchman 1995; Monaco 2008).

As to point 5, more remains to be said. Leonardo da Vinci was born in one of the richest areas for ichnofossils in the Italian Peninsula, corresponding to the Apennine foredeep deposits. These are mainly represented by siliciclastic turbidites (Oligo-Miocene) associated with a diverse deep-sea ichnofauna (Uchman 1995). *Paleodictyon* is particularly common within the Marnoso-Arenacea Formation (Monaco 2008), which crops out in a vast area around Florence. In the years prior to the drafting of Codex I and the Leicester Codex, Leonardo traveled extensively through these ichnofossil-rich deposits, as witnessed by his biographical data (Kemp 2006; Zöllner 2007). Leonardo even represented the Apennine foredeep deposits in some of his paintings (*The Virgin and Child with Saint Anne; Baptism of Christ;* Vai 1995, 2003), whose rocky outcrops share the most typical characteristics of the Marnoso-Arenacea Formation.

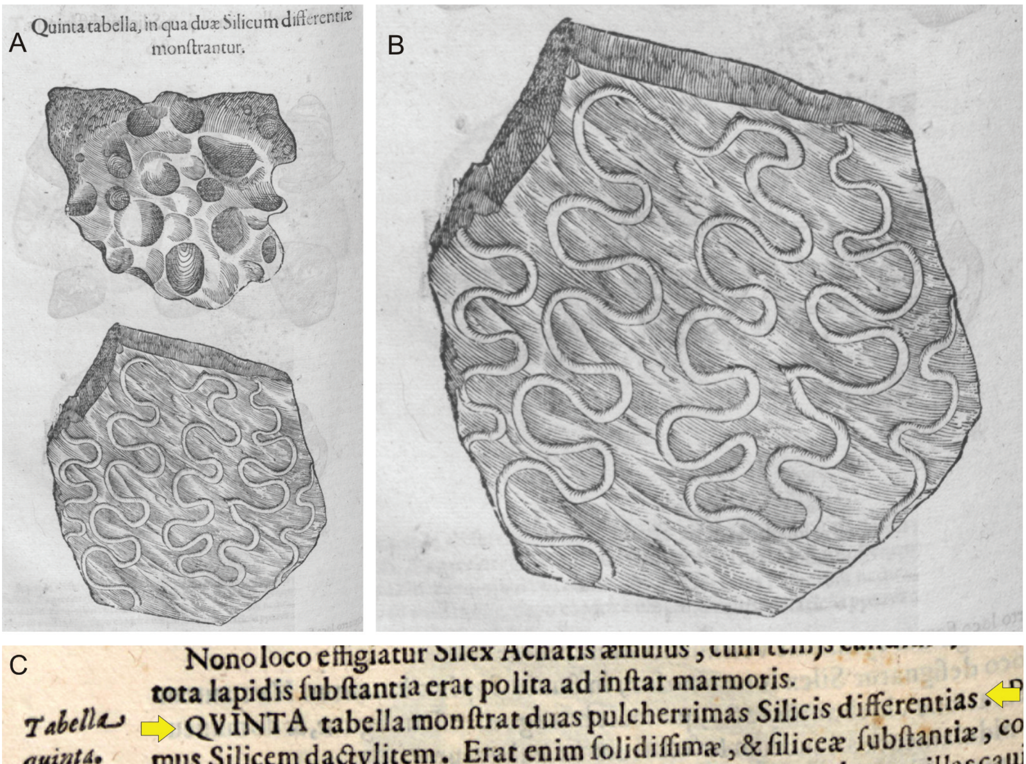
Trace fossils are *pulcherrimas* (very beautiful)

As stated before, Leonardo was not the only naturalist to depict trace fossils. Seilacher (2007) cited the Swiss naturalist Johann Bauhin, who gave a fanciful description of *Phymatoderma*. To Bauhin the intricate branches of this trace fossil represented angelic figures. Chondritids were probably discussed also in Gesner's *De omni rerum fossilium genere,* in which a specimen is described as "dark fissile rock in which are branches impressed by nature" (*lapis fissilis niger, in quo rami impressi à natura*). Another significant example is that of Ulisse Aldrovandi, the naturalist who coined the term 'geology' (Vai and Cavazza 2003). Aldrovandi discussed at length about body and trace fossils in his geo-palaeontological treatise *Musaeum Metallicum* (Text-fig. 3; Baucon 2008a, b, 2009b).

Among the wonderful illustrations that accompany the *Musaeum Metallicum,* there is one that includes both a bioturbation trace (*Cosmorhaphe*) and a bioerosion trace (*Gastrochaenolites)*. In describing these traces, Aldrovandi said (Text-fig. 4c):



Text-fig. 3. Aldrovandi anticipated certain aspects of the Galilean revolution, but he also resorted to notions that are scientifically backward-looking. A – Aldrovandi desired to gather all he could find amazing or unusual in nature, including fossils but also fantastic creatures and freaks. He commented on this illustration as a "winged and horned monster, similar to a Cacodemon." B – Aldrovandi described this fossil as "rock pregnant with a shell". In reality it is a fossil bivalve. C – Aldrovandi called fossil crinoids *Astroitis,* a word that comes from the Latin *aster,* meaning 'star'. D – Aldrovandi named fossilized shark teeth *Glossopetrae* or 'tongue-like stones'



Text-fig. 4. Trace fossils from Aldrovandi's *Musaeum Metallicum.* A – *Cosmorhaphe* and *Gastrochaenolites*. B – Detail of A, showing *Cosmorhaphe.* Note the use of hatching to restore three-dimensionality and volume to the subject. C – Caption of the plate with *Cosmorhaphe* and *Gastrochaenolites.* Aldrovandi describes these traces as "very beautiful" (arrowed sentence) "the fifth plate shows two very beautiful varieties of flint".

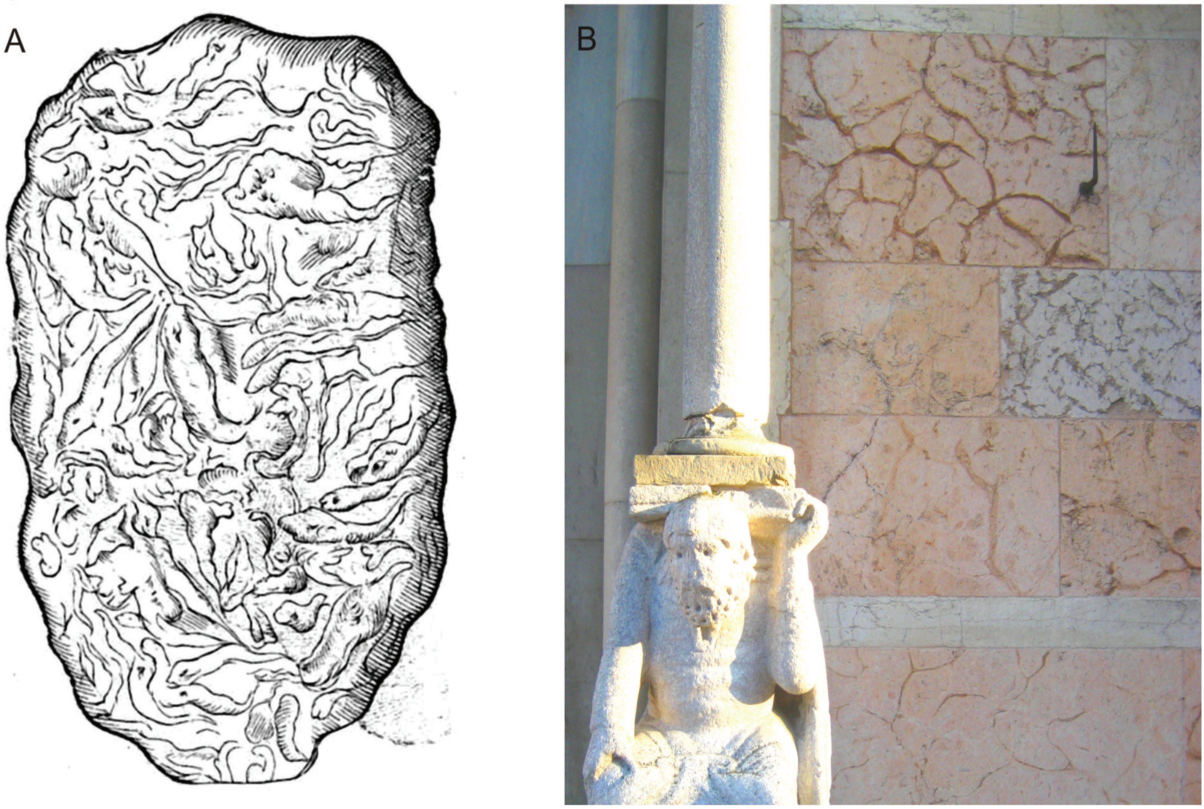
Quinta tabella monstrat duas pulcherrimas Silicis differentias.

(Aldrovandi, *Musaeum Metallicum,* p. 730)

The use of the Latin superlative *pulcherrimas* leaves no doubt to the aesthetic appreciation for these trace fossils. As to the use of *silex* (literally, 'flint'), it has been shown that Aldrovandi used the term to indicate different rocks, including certain kinds of limestone and sandstone (Baucon 2009b).

Aldrovandi misinterpreted the nature of *Cosmorhaphe* as he described it as a natural curiosity imitating the sinuous curves of a snake. Aldrovandi referred to snakes also in portraying the typical ichnofabric of the 'Verona Stone' (Text-fig. 5b), an ornamental stone celebrated in Italy from the Renaissance onwards. 'Verona Stone' is extracted from the pelagic nodular limestones of the Rosso Ammonitico Formation (Jurassic), which is typically characterized by moderate to intense bioturbation, often dominated by *Thalassinoides* (Monaco 1995). Rosso Ammonitico Formation bears abundant cephalopods, as testified by Leonardo himself: "In the mountains around Verona there is its red stone full of shells" (*Truovasi nelle montagne di Verona la sua pietra rossa mista tutta di nichi,* Leicester Codex 9v; cited by Ligabue 1977).

In his *Musaeum Metallicum* Aldrovandi correctly interpreted bioerosional traces, as witnessed by a specimen described as "pitted here and there by hollows of varying size" (*Erat* [...] *passim sinubus diversae magnitudinis excavatus*). The corresponding illustration (Text-fig. 4a, top) reveals the ichnogenus *Gastrochaenolites,* a bioerosional trace commonly produced by bivalves (Baucon 2009b). Aldrovandi described those borings as "hollows [...] resembling the cavities in which piddocks [*Pholas dactylus*] seek shelter" (*qui imitabantur illas cavitates, in quibus solent Dactyli animantes delitescere solent*). Bioerosional structures are also dealt in Chapter L of the *Musaeum Metallicum,* entitled "De Lapide Pholadis" ("On Rock with Pholads"). Intriguingly, the naturalist Konrad Gesner, an inspiration for Aldrovandi, represented borings too (Text-fig. 6b). In addition to *Cosmorhaphe* and *Gastrochaenolites,* Aldrovandi depicted other trace fossils: *Thalassinoides*, some fragmentary burrows (possibly *Thalassinoides* or *Planolites* / *Palaeophycus;* Text-fig. 6c) and a doubtful specimen of *Chondrites*.



Text-fig. 5. Illustrating ichnofabric. A – Aldrovandi used snakes to represent the bioturbated fabric of the 'Verona Stone', an ornamental stone extracted from the Rosso Ammonitico Formation. Note the analogy with the texture of the 'Verona Stone' of the Piacenza Cathedral (B)

AESTHETIC AND COGNITIVE PSYCHOLOGY OF TRACES

The fractal dimension as a measure of the aesthetic value of traces

Ichnology began as an aesthetic appreciation of invertebrate traces, a taste that is also represented by contemporary artists.

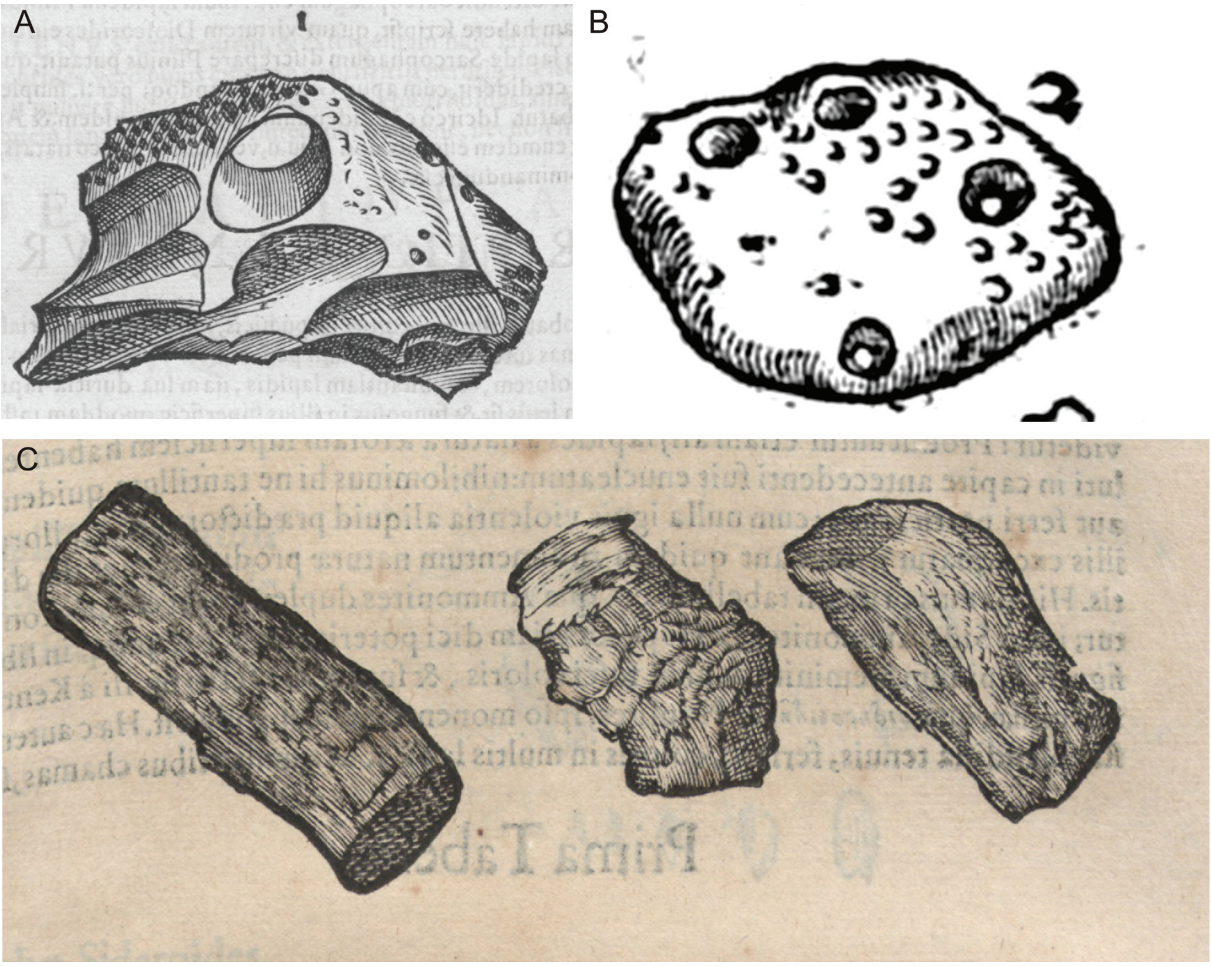
Some questions arise from these points:

1. Why do certain traces stimulate the psychological experience of beauty?
2. What psychological mechanisms lie behind the aesthetic appreciation of traces?

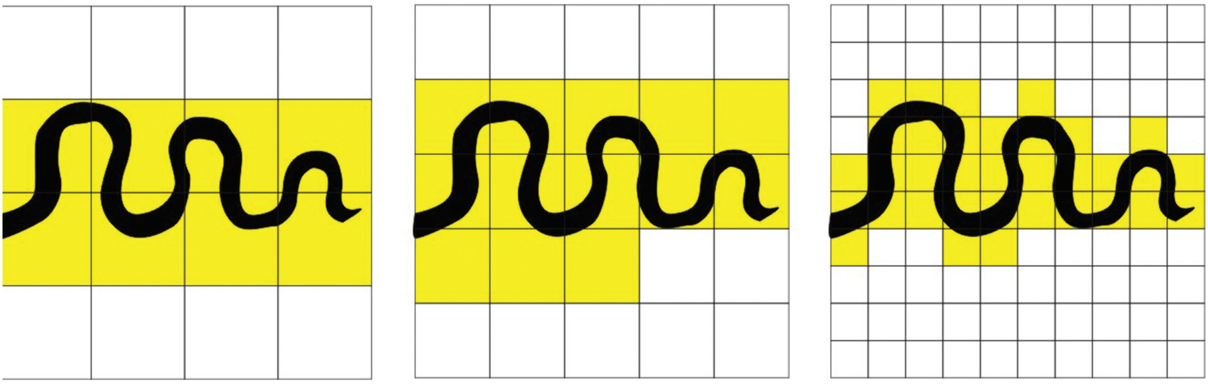
To answer these questions, we have to resolve another issue: What universal elements make a form attractive? The answer to this question has been sought since the days of Plato, followed by many other philosophers (e.g. Aristotle, Kant, Kames, Burke, Hogarth); in more recent times, several researchers approached the problem mathematically (Fechner 1876; Birkhoff 1933; Davis 1936; Rashevsky 1938; Eysenck 1941). Among analytical methods, fractal analysis proved to be the most efficient both in quantifying visual attractiveness and in describing the structure of complex patterns (Mandelbrot 1983; Sprott 1993; Sphear *et al*. 2003). In fact, fractal geometry has been applied in studying visual perception itself (Pentland 1984; Cutting and Garvin 1987; Sprott 1993; Sphear *et al*. 2003), and has been used to analyse abstract art (Taylor *et al*. 1999), architecture and design (Bovill 1995).

In general terms, a fractal is "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole" (Mandelbrot 1983).

Since their discovery, fractals have had considerable success in quantifying the complex structure exhibited by many natural patterns (Sphear *et al*. 2003).



Text-fig. 6. More trace fossils from the Renaissance. A – *Gastrochaenolites* depicted in Aldrovandi's *Musaeum Metallicum*. B – Borings illustrated in Gesner's *De omni rerum fossilium genere*. C – Aldrovandi, fragmentary burrows

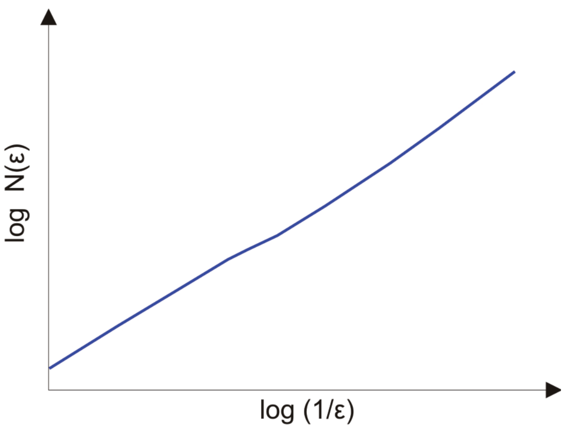


Text-fig. 7. Box-counting method. The image is covered by a quadratic grid of progressively smaller mesh. The method counts the number of boxes N(ε) containing a segment of trace for different box sizes (ε)

'Traditional' (Euclidean) geometry has difficulty in measuring complex and irregular shapes.

We are trained to use integral dimensions (i.e. one-dimensional lines, two-dimensional squares, three-dimensional cubes) but many things are better described with fractionary (fractal) dimensions. In fractal geometry, the fractal dimension D describes how the patterns occurring at different magnifications combine to build the resulting fractal shape (Gouyet 1996). In other words, it describes an object by quantifying the scaling relation among patterns observed at different magnifications.

It has been shown that there is a close relationship between the fractal dimension and the visual attractiveness of a form (Pentland 1984, Cutting and Garvin 1987). More in particular, Sprott (1993) presented about 7500 fractal images to eight observers who rated them on a five-point scale for their aesthetic appeal. The author concluded that images with a fractal dimension between 1.1 and 1.5 are considered to be the most aesthetically appealing. A similar experiment by Aks and Sprott (1996) agreed with the previous results, highlighting the fractal dimension of 1.3 as the most appealing. In their psychological studies on visual perception, Sphear *et al*. (2003) tested images generated by natural processes, by mathematics, or by the human hand. Higher preference was given for images with a fractal dimension between 1.3 and 1.5, regardless of the images' origin.



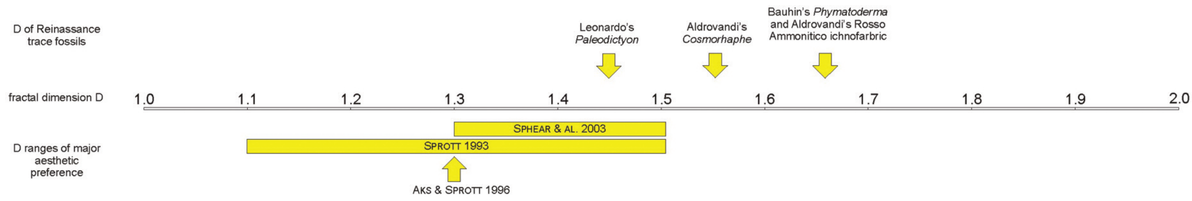
Text-fig. 8. Fractal dimension. If we plot log (1/ ε) against log N(ε), the fractal dimension is given by the gradient of the best fit line

I estimated the fractal dimension of the ichnological drawings of the Renaissance by using the box-counting method (Mandelbrot 1983; Sphear *et al*. 2003). This method consists of covering the image with a quadratic grid (Text-fig. 7) and counting the number of boxes N(ε) containing a segment of trace for different box sizes (ε). If log (1/ ε) is plotted against log N(ε), the gradient of the line of best fit equals the limit of log N(ε) / log (1/ε) for ε→0; that is, the fractal dimension (Text-fig. 8).

The method revealed 'very appealing' fractal dimensions for Leonardo's *Paleodictyon* (D = 1.48) and Aldrovandi's *Cosmorhaphe* (D = 1.56), which fall in the range of maximum visual preference of Sprott (1993) (D = 1.1–1.5; Text-fig. 9) and Sphear *et al*. (2003) (D = 1.3–1.5). Slightly higher fractal dimensions are displayed by Aldrovandi's ichnofabric (D = 1.67) and Bauhin's *Phymatoderma* (D = 1.64). Drawings of fragmentary burrows and borings have been excluded from the analysis, as their estimated fractal dimension would be trivial.

Traces as fractals: the key of the aesthetics of ichnology

In the previous section I showed that many traces are attractive because they present intermediate fractal dimensions. Nevertheless, aesthetic preference is not simply a function of the fractal dimension but depends also on the (fractal) organization of patterns. Indeed, several studies have confirmed that



Text-fig. 9. Fractal dimensions of ichnological drawings from the Renaissance

humans display a consistent aesthetic preference across fractal images (Peitgen and Richter 1986; Shearer 1992; Flake 1999; Taylor *et al*. 1999; Sphear *et al*. 2003; Taylor 2006). Consequently, one must wonder if some traces have a fractal-like organization. To answer this, two complementary approaches are proposed:

1. In the first approach, the empirical, I attempt to replicate existing trace fossils using L-systems;
2. In the second approach, the analytical, I evaluate whether some trace fossils have fractal features.

Traces described by L-systems

L-systems were so called by the Hungarian botanist Aristid Lidenmayer, who introduced them to model cell development and plant branching. They are considered to be a very simple way to generate fractals, and Plotnick (2003) has already utilised them to reproduce different ichnogenera (though without referring to their fractal nature).

An L-system defines complex objects by successively replacing parts of a simple object (axiom) by using a set of rewriting rules. Rewriting can be carried out for *n* steps

Here is an example (Table 1):

Imagine now that the symbols are instructions for drawing geometrical patterns with a relative cursor – i.e. moving with commands that are relative to its own position. *F* means 'draw forward', '+' turn right (without drawing), and '–' 'turn left (without drawing)'.

By applying these simple rules, it is possible to obtain complex fractals such as the Koch curve (Text-fig. 10), a famous fractal with infinite length. With the same rules, one can obtain different ichnogenera.

It should be noted that L-systems are used here to investigate the geometry of traces, not to find a physical mechanism for their production. The rewriting rules are not intended as a set of behavioural ones; to the contrary, the focus is on traces as geometric patterns described by iterative rewriting rules. In fact, as noted by Plotnick (2003), an alternative way to describe L-system graphics is that one part of a geometric figure is replaced by another geometric figure, with the replacement occurring iteratively. Therefore the aim is to show that various traces can be conceived as approximate fractals composed of self-similar sections. This approach has been used for replicating the patterns of *Paleodictyon, Palaeomeandron*, *Chondrites, Gordia* and *Cosmorhaphe* (Text-figs 11, 12). Notably, L-systems can be tested even in three dimensions (as well as demonstrated by the ichnogenera simulated by Plotnick 2003). In the case of

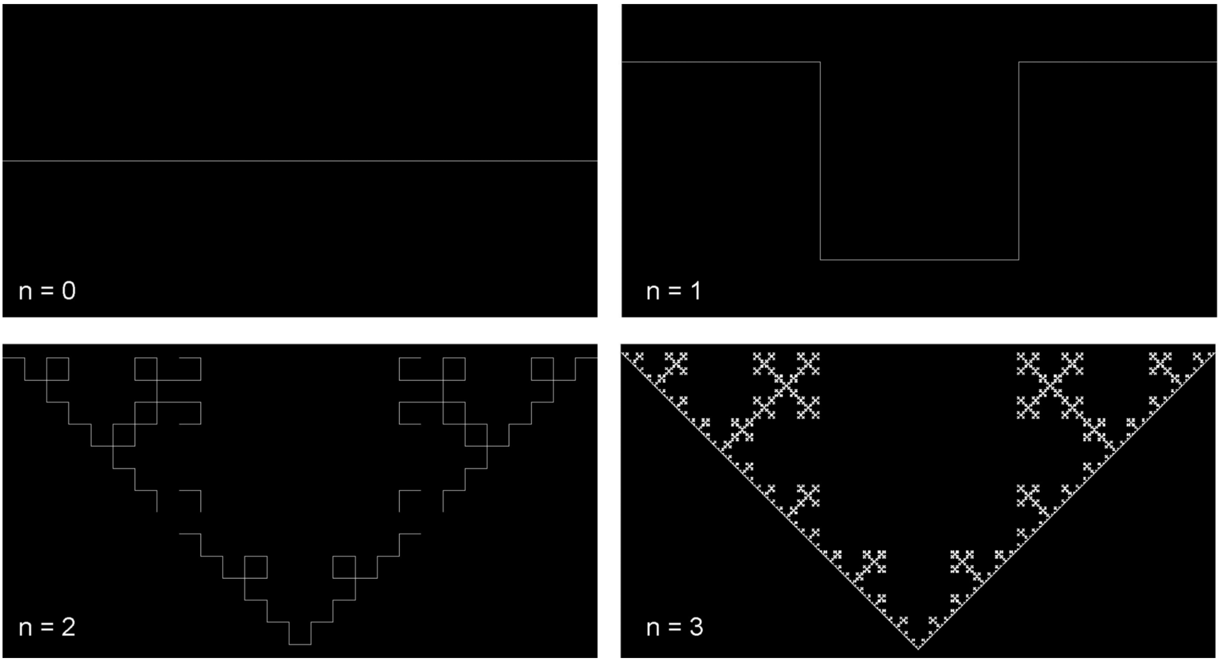
symbols: F, +, -

axiom: F

rewriting rule: each F is substituted by the string 'F+F--F+F'

Step (n)	
0	F
1	F+F--F+F
2	F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F
3	F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F+F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F--F+F- -F+F+F+F--F+F--F+F--F+F+F+F--F+F+F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F

Table 1. An L-system



Text-fig. 10. Koch curve generated by the L-system in the previous table. The turn angle is 60°

'L-*Paleodictyon'* in Text-fig. 11a, note that the outline of the whole system is self-similar to each hexagonal mesh. This detail has also been observed in many fossil and recent *Paleodictyon* (e.g., see Seilacher 2007, p. 157, 158)

The application of L-systems demonstrates that:

1. Various traces can be intuitively described as (approximate) fractals composed of self-similar parts.
2. Real-world traces are approximate fractals. For instance, the Koch curve is obtained by applying a construction rule for an infinite number of times, which leads to self-similarity over all scales. At the contrary, real-world traces are fractal over a limited range of scales, as exemplified by the limited number of iterations for describing their morphology.

Fractal behaviour of some trace fossils

The concept of fractal is associated with geometrical objects satisfying (1) fractional dimensionality (Euclidean objects have integer dimensions) and (2) self-similarity (Peng *et al*. 1999).

By enlarging a fractal object an increasing number of self-similar details is obtained. Borrowing a classic example from Mandelbrot (1983), if one were to measure the coastline of Britain with a long yardstick, one would get a shorter result than by measuring with a small ruler. The central concept is that the length and size of measurement are related by a power law.

A common power law demonstrates the internal self-consistency of the fractal and its unity across scales (Mandelbrot 1983; Schroeder 1995). However, self-similarity should hold on all scales for mathematical fractals only, while in the real world there are necessarily lower and upper bounds over which such behaviour applies (Peng *et al*. 1999; Vicsek 2001). According to Kaandorp (1994) and Vicsek (2001), if an irregular biological pattern shows the property of self-similarity within a certain interval of scales it has fractal behaviour.

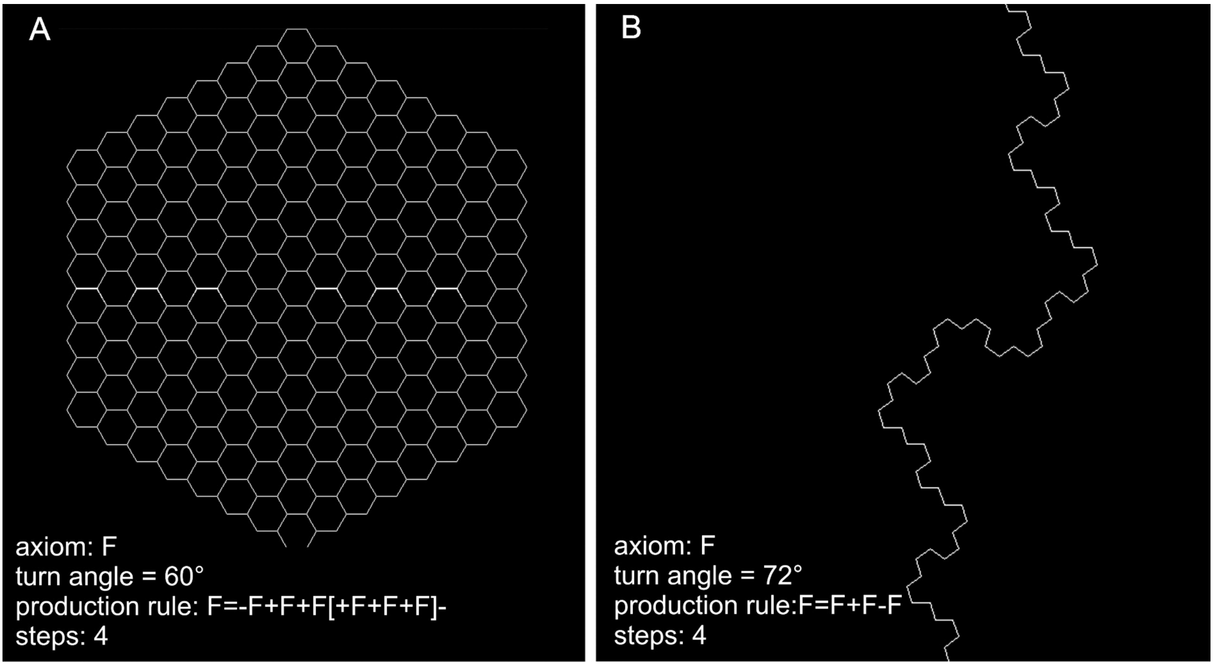
On the basis of these elements, if the curve obtained with the box-counting method does not fit a power law, two conclusions are possible: Either the trace is not of a fractal nature or it is of a multi-fractal nature (Text-fig. 13; Table 2).

On the grounds of the aforementioned theoretical framework, I tested the fractal behaviour of the ichnological drawings from the Renaissance. I found fractal behaviour in *Paleodictyon* (by Leonardo), *Cosmorhaphe* and Rosso Ammonitico ichnofabric (by Aldrovandi).

I tested the same methods on other trace fossils. Interestingly, some of the most figured traces of the Renaissance – graphoglyptids and chondritids – reveal fractal behaviour.

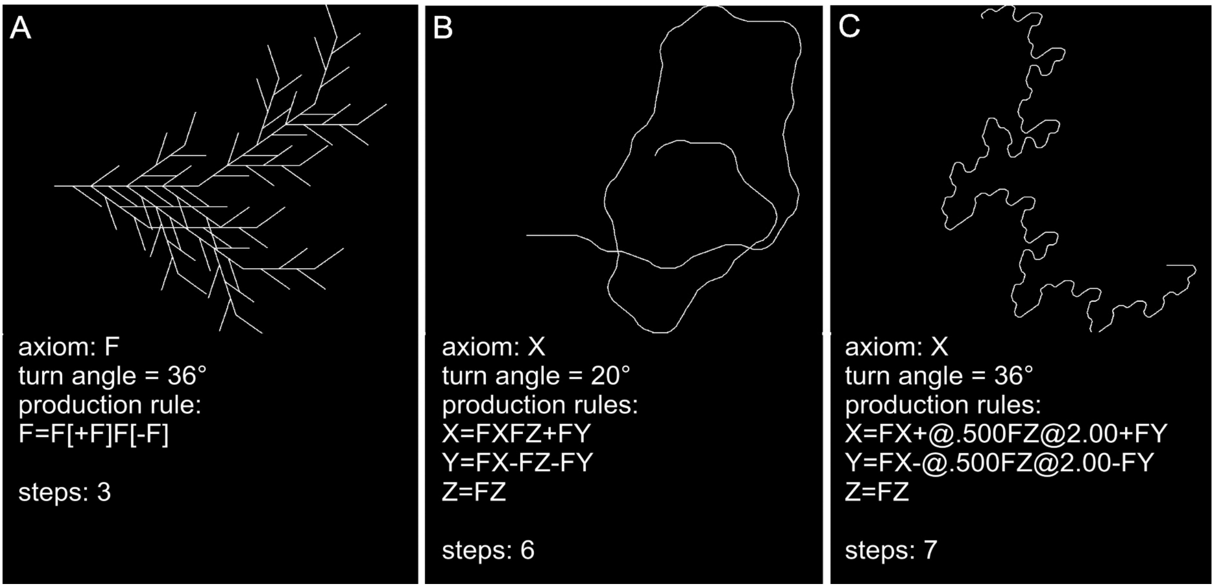
In particular, the scaling behaviour and fractal dimensions of graphoglyptids prove:

1. Graphoglyptids are aesthetically attractive patterns for their intermediate fractal dimensions (most of the examined specimens present a fractal dimension between 1.5 and 1.6; Text-fig. 14) and for their fractal behaviour. Being rich in structure, these fractal traces attract visual interest.



Text-fig. 11. Traces generated by 'two-dimensional' L-Systems. A — *Paleodictyon*. B — *Palaeomeandron*

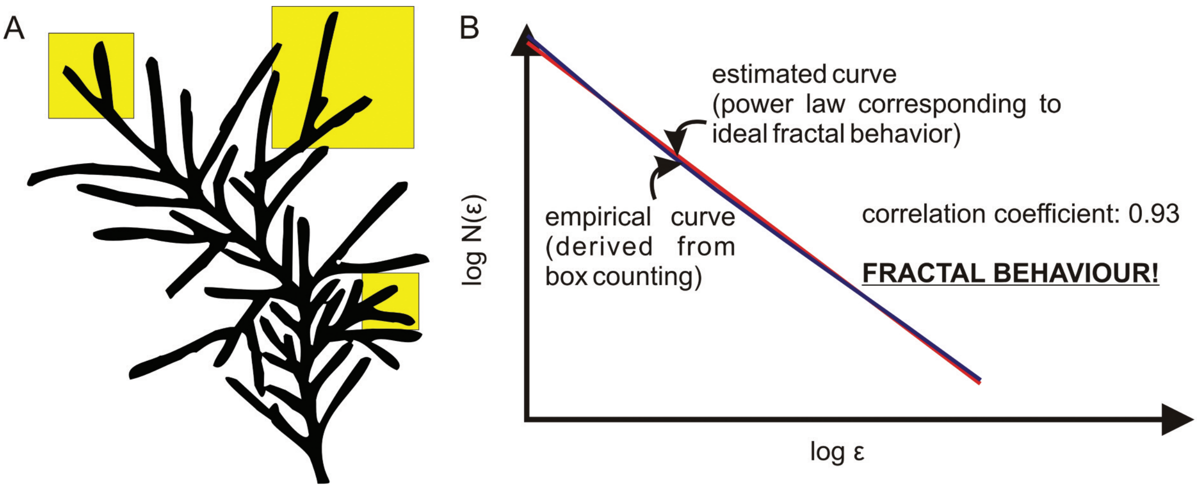
2. Graphoglyptids cover the space, but not completely. Rather, the fractal dimension can be conceived as a statistical quantity that gives an indication of how completely a pattern fills space, as one zooms down to finer and finer scales; for instance, a fractal dimension of 2 means the complete filling of the plane. In graphoglyptids, intermediate fractal dimensions are probably related to their constructional program and function. Many graphoglyptids may be farming burrows and for this reason do not cover a given surface, but subdivide it into areas of similar dimensions (Seilacher 2007, p. 150). This leads to fractal behaviour and intermediate fractal dimensions, while foraging traces (e.g. *Helminthoida*) have higher fractal dimensions.



Text-fig. 12. Traces obtained as L-systems. A – *Chondrites*. B – *Gordia*. C – *Cosmorhaphe*. The grammar includes the following symbols: [and] control branching rules, as '['means "remember the current graphics state" and " ']' means 'set the graphics state back to the remembered state'. '@' means 'multiply the current line length by the amount following'. X, Y, Z do not correspond to any drawing action but are only used to control the evolution of the curve

Step	Description
1	The image is covered by a quadratic grid.
2	We count the number of boxes N(ε) containing a segment of trace for a given box size (ε).
3	The box-counting (previous point) is repeated for progressively smaller box sizes (ε).
4	The empirical curve is obtained by plotting ε against N(ε).
5	The next stage is to fit the empirical curve from counting method, with another one, the estimated curve. Because a trace (as well as natural images) is not a pure fractal (it is not a continuous function but a discrete and finite one), it is only possible to approximate the fractal law (estimated curve, obtained with linear or logarithmic regression).
6	If the empirical curve follows a fractal law, the estimated curve has the form of a power law (parabolic or hyperbolic). $N = \varepsilon^{D}$ or $N = \varepsilon^{-D}$.
7	Moreover, if the empirical curve does not fit the estimated one, the trace is of a nonfractal or multifractal nature.

Table 2. How to recognize the fractal behaviour of a trace



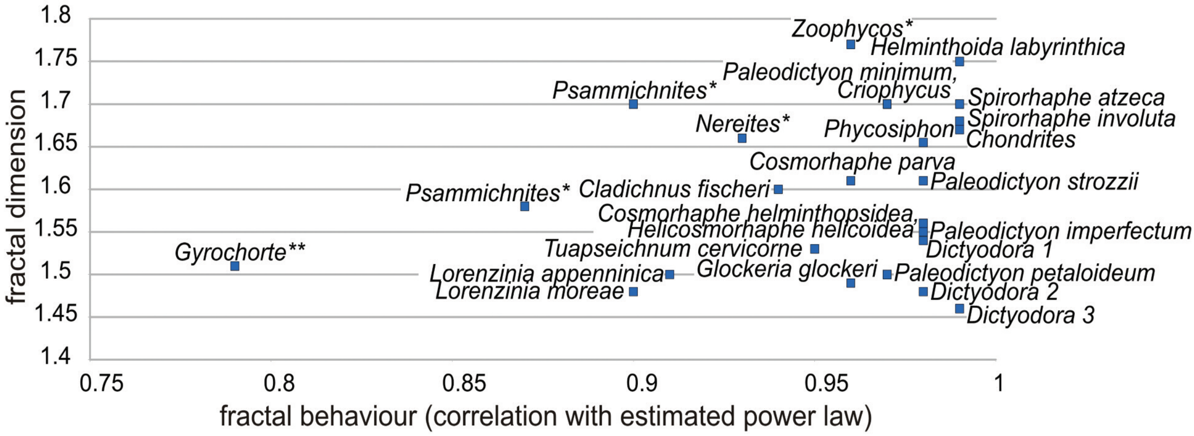
Text-fig 13. How to recognize fractal behaviour in traces. A – Intuitively, *Chondrites* is a self-similar object: there are several orders of branching at different scales (highlighted). Drawing based on Seilacher (2007), p. 143. B – The 'intuitive' approach confirmed by the high correlation between the estimated and the empirical curve: A power law determines the self-similarity of the structure

DISCUSSION

The fractal beauty of traces

In sum, some of the most emblematic figures of the Renaissance – da Vinci, Gesner, Aldrovandi, Bauhin – were pioneers of ichnology and demonstrated a visual interest in trace fossils. To decipher the reason for this aesthetic appreciation, I used fractal geometry to explore various ichnological drawings of the Renaissance. Recent studies on visual perception (Sprott 1993; Sphear *et al*. 2003) proved that humans have an aesthetic preference for fractal objects and images with intermediate fractal dimension. These paradigms are found not only in the ichnological drawings from the Renaissance but also in many trace fossils. In particular, graphoglyptids and chondritids meet these aesthetic paradigms and, not surprisingly, are among the most figured trace fossils of the Renaissance. Being rich in structure, such traces have been acknowledged by naturalists for their instant aesthetic appeal.

Consequently, one might suppose that fractal beauty could influence the direction of modern ich-



Text-fig. 14. Fractal dimension and fractal behaviour of graphoglyptids and other traces. The measured specimens come from Seilacher (2007) and from the Italian Alps (* = Pramollo, Carboniferous; ** = Sauris, Triassic)

nology as well. Do structured (fractal) morphologies receive more scientific attention than simple ones? Some cases suggest a positive answer (i.e. graphoglyptids received a significant attention), but further studies are required to answer to such a question.

Moreover, the above analysis shows that fractal geometry is effective in describing the morphology of traces, hence it could be an excellent tool for different aspects of ichnological research. Despite the fact that traces are complex and irregular, until now fractal geometry has not been widely used by ichnologists (Jeong and Ekdale 1996; Gibert *et al.* 1999; Neto de Carvalho 2001, 2003, 2004, 2006; Neto de Carvalho and Cachão 2006).

Further perspectives: fractal traces as self-organized extended organisms

In this study it emerged that several traces have fractional dimensionality and self-similarity over a limited range of measurement scales (fractal behaviour). This observation is corroborated by the successful application of L-systems that describe traces as approximate fractals constituted by self-similar sections.

These 'fractal traces' are hierarchically structured landscapes and the emergence of order is fundamentally based on correlations between different scales. A fractal structure exhibits complex geometry developed from simple rules and thus is a manifestation of the universality of self-organization processes. In fact, self-similarity behaviour is typical of self-organizing systems (Camazine *et al*. 2003).

If certain traces are fractal and self-organized, they could then be regarded as an emergent property of the tracemaker. This assumption is connected with the theory of the extended organism, considering that certain traces are external organs of physiology, channelling or tapping into energy sources for doing physiological work (Turner 2003). For example, *Chondrites* exemplifies a structure that is fractal and self-organized, hence conceivable as part of an extended organism. The same consideration is valid for other ichnogenera, such as *Paleodictyon* and *Cosmorhaphe*.

On the other hand, the concept of extended-organism implies a fractal nature of the system in which the "physiological boundaries can exist at multiple levels, and these nest hierarchically, like Matryoshka dolls" (Turner 2003). Considering certain fractal traces as organs of extended physiology, then the tracemaker and its traces could be linked by a fractal self-similarity. Therefore, it is suggested that fractals would allow reverse engineering of traces by exploring the hierarchy rules governing the organization of structures at each level of scale. This approach should help us to understand function from structure of traces, and to answer questions about how physiology relates to morphology.

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